CHAPTER 13

SURFACE AREAS AND VOLUMES

MATHEMATICAL REASONING

1. If the height of a cylinder is doubled, by what number must the radius of the base be multiplied so that the resulting cylinder has the same volume as the original cylinder?
(A) 4 (B) $\frac{1}{\sqrt{2}}$
(C) 2 (D) $\frac{1}{2}$

2. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted into a cube. Find the difference between surface areas of two solids.
(A) 280 cm^2 (B) 284 cm^2
(C) 296 cm^2 (D) 286 cm^2

3. The height of a cone is equal to its base diameter. Then slant height of the cone is
(A) $\sqrt{r^2+h^2}$ (B) $r\sqrt{5}$
(C) $h\sqrt{5}$ (D) $rh\sqrt{5}$

4. The length of the longest rod that can be kept in a cuboidal room of dimensions 10 m × 10 m × 5 m is _____.
(A) 16 m (B) 10 m
(C) 15 m (D) 12 m

5. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, then the amount of the beverage that can be poured from the bowl into the cylindrical vessel is _____.
(A) $66\frac{2}{3}\%$
(B) $78\frac{1}{2}\%$
(C) 100%
(D) None of these

6. The volume of a cylinder of radius r is 1/4 of the volume of a rectangular box with a square base of side length x. If the cylinder and the box have equal heights, what is the value of r in terms of x?
(A) $\frac{x^2}{2\pi}$ (B) $\frac{x}{2\sqrt{\pi}}$
(C) $\frac{\sqrt{2x}}{\pi}$ (D) $\frac{x}{\sqrt{\pi}}$

7. The edge of a cube is 20 cm. How many small cubes of edge 5 cm can be formed from this cube?
(A) 4 (B) 32 (C) 64 (D) 100

8. The volume of two spheres are in the ratio 64 : 27. The difference of their surface areas, if the sum of their radii is 7 units, is _____.
(A) 28π sq. units (B) 88 sq. units
(C) 88π sq. units (D) 4π sq. units

9. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is _____.
(A) 10 : 17 (B) 20 : 27
(C) 17 : 27 (D) 20 : 37

EVERYDAY MATHEMATICS

10. A covered wooden box has the inner measures as 115 cm, 75 cm, 35 cm and the thickness of wood is 2.5 cm. Then the volume of the wood is ______.
(A) 80000 cu. cm (B) 82125 cu. cm
(C) 84000 cu. cm (D) 85000 cu. cm

11. A spherical ball of lead, 3 cm in radius is melted and recast into three spherical balls. The radius of two of these are 1.5 cm and 2 cm respectively. The radius of the third ball is ______.
(A) 2.66 cm (B) 2.5 cm
(C) 3 cm (D) 3.5 cm

12. How many metres of cloth, 5 m wide, will be required to make a conical tent, the radius of whose base is 7 m and height is 24 m?
(A) 550 m (B) 168 m
(C) 110 m (D) 33.6 m

13. Water flows in a tank 150 m × 100 m at the base, through a pipe whose cross-section is 2 dm by 1.5 dm at the speed of 15 km per hour. In what time, will the water be 3 metres deep?
(A) 50 hours (B) 150 hours
(C) 100 hours (D) 200 hours

14. A teak wood log is first cut in the form of a cuboid of length 2.3 m, width 0.75 m and of a certain thickness. Its volume is 1.104 m^3. How many rectangular planks of size 2.3 m × 0.75 m × 0.04 m can be cut from the cuboid?
(A) 16 (B) 64 (C) 68 (D) 4

15. A circus tent is cylindrical to a height of 3 metres and conical above it. if its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.
(A) 1996 m (B) 2096 m
(C) 1947 m (D) 1800 m

16. A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height of 12 cm, then how many litres of milk is needed to serve 1600 students?
(A) 739.2 litres (B) 538 litres
(C) 740 litres (D) 400 litres

17. A small village, having a population of 5000, requires 75 litres of water per head per day. The village has got an overhead tank of measurement 40 m × 25 m × 15 m. For how many days will the water of this tank last?
(A) 30 days (B) 32 days
(C) 40 days (D) 45 days

ACHIEVERS SECTION (HOTS)

18. Read the statement carefully and write 'T' for true and 'F' for false.
(i) Volume of a cylinder is three times the volume of a cone on the same base and of same height.
(ii) Volume of biggest sphere in cube of edge 6 cm is 36π cm^3.
(iii) Cuboids and cubes are special forms of right prisms.

	(i)	(ii)	(iii)
(A)	T	F	T
(B)	T	T	T
(C)	F	T	F
(D)	F	T	T

19. Match the following.

Column-I	Column-II
(P) A cylinder of radius 3 cm is inscribed in a sphere of radius 5 cm, then volume of cylinder is ______.	(1) 38.5 cm^3
(Q) A conical pit of top diameter 3.5 cm is 12 cm deep, the capacity of pit is ______.	(2) 512 cm^3
(R) The length of a diagonal of a cube is $8\sqrt{3}$ cm, then volume of cube is ______.	(3) 72π cm^3
(S) The capacity of a conical vessel with height 12 cm and slant height 13 cm is ______.	(4) 100π cm^3

(A) (P)→(2) ; (Q)→(3) ; (R)→(4) ; (S)→(1)

(B) (P)→(1) ; (Q)→(3) ; (R)→(2) ; (S)→(4)

(C) (P)→(3) ; (Q)→(1) ; (R)→(2) ; (S)→(4)

(D) (P)→(4) ; (Q)→(1) ; (R)→(3) ; (S)→(2)

20. Study the statements carefully.

Statement-I : If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Statement-II : Curved surface area is increased when diameter decreases.

Which of the following options hold?

(A) Both Statement-I and Statement-II are true.

(B) Statement-I is true but Statement-II is false.

(C) Statement-I is false but Statement-II is true.

(D) Both Statement-I and Statement-II are false.

☺☺☺

13 Surface Areas and Volumes

1. (B) : Let radius and height of original cylinder be r_1 and h_1 respectively

$\therefore$ Volume of original cylinder $= \pi r_1^2 h_1$

Also, let radius of new cylinder be r_2 and height of new cylinder

$= 2 \times$ (height of original cylinder)

$= 2 \times h_1 = 2h_1$

$\therefore$ Volume of new cylinder $= \pi r_2^2 \cdot 2h_1$

According to question,

Volume of original cylinder = Volume of new cylinder

$$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2 \cdot 2h_1 \Rightarrow r_1^2 = 2r_2^2 \Rightarrow r_2 = \frac{1}{\sqrt{2}} r_1$$

Hence, radius of base of original cylinder must be multiplied by $\frac{1}{\sqrt{2}}$ so that the new cylinder has same volume as original.

2. (D) : Volume of the metal sheet

$= (27 \times 8 \times 1)\ cm^3 = 216\ cm^3$

Surface area of the metal sheet

$= 2(27 \times 8 + 8 \times 1 + 1 \times 27)$

$= 2(216 + 8 + 27) = 502\ cm^2$

Volume of cube = Volume of metal sheet

$(side)^3 = 216\ cm^3 \Rightarrow side = 6\ cm$

Now, surface area of cube $= 6(6)^2 = 216\ cm^2$

$\therefore$ Required difference $= (502 - 216)\ cm^2 = 286\ cm^2$

3. **(B)** : Let radius of the cone = r

$\therefore$ Height of the cone (h) = diameter = $2r$

$\therefore$ Slant height of the cone (l) = $\sqrt{h^2 + r^2}$

$= \sqrt{(2r)^2 + r^2} = \sqrt{5r^2} = \sqrt{5}\,r$

4. **(C)** : Given, length of cuboid (l) = 10 m

Breath of cuboid (b) = 10 m

Height of cuboid (h) = 5 m

As, length of longest rod in cuboid = Diagonal of cuboid

$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(10)^2 + (10)^2 + (5)^2}$

$= \sqrt{100 + 100 + 25} = \sqrt{225} = 15$ m

5. **(C)** : Let radius of hemispherical bowl and cylindrical vessel be r.

Also, r = 50% more than h

$\Rightarrow r = 50\%$ of $h + h$

$\Rightarrow r = \frac{3h}{2}$ ...(i)

Now, volume of bowl $(V_1) = \frac{2}{3}\pi r^3$ ...(ii)

and volume of vessel $(V_2) = \pi r^2 h$ ...(iii)

Dividing eqn. (iii) by (ii), we get

$\frac{V_1}{V_2} = \frac{\frac{2}{3}\pi r^3}{\pi r^2 h} = \frac{2r}{3h} = \frac{2}{3h}\left(\frac{3h}{2}\right)$ [by (i)]

$= 1$

$\Rightarrow V_1 = V_2$

$\therefore$ Volume of bowl = Volume of vessel

The amount of beverage that can be poured into vessel is 100%.

6. **(B)** : Let the height of cylinder and rectangular box be h.

Volume of cylinder = $\pi r^2 h$

$\therefore$ Volume of rectangular box $= x \times x \times h = x^2 h$

According to question,

Volume of cylinder = $\frac{1}{4}$ × Volume of rectangular box

$\Rightarrow \pi r^2 h = \frac{1}{4} \times x^2 h \Rightarrow r^2 = \frac{x^2}{4\pi}$ or $r = \frac{x}{2\sqrt{\pi}}$

7. **(C)** : Let 'n' be the number of cubes which can be formed from the given cube.

Volume of big cube = Volume of n smaller cubes

$\Rightarrow 20 \times 20 \times 20 = n \times 5 \times 5 \times 5$

$\Rightarrow n = \frac{20 \times 20 \times 20}{5 \times 5 \times 5} = 4 \times 4 \times 4 = 64$

8. **(A)** : Let r_1 and r_2 be radii of two spheres. According to question,

$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27} \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{64}{27} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$ ...(i)

Given, $r_1 + r_2 = 7$ ...(ii)

From (i) and (ii), we get $r_1 = 4$ units, $r_2 = 3$ units

$\therefore$ Required difference $= 4\pi r_1^2 - 4\pi r_2^2$

$= 4\pi(4^2 - 3^2) = 4\pi \times 7 = 28\pi$ sq. units.

9. (B) : Let r_1, r_2 be the radius of two cylinders

$\therefore \quad \frac{r_1}{r_2} = \frac{2}{3}$

Let h_1 and h_2 be height of two cylinders

$\therefore \quad \frac{h_1}{h_2} = \frac{5}{3}$

Now, $\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \cdot \frac{h_1}{h_2} = \left(\frac{2}{3}\right)^2 \cdot \frac{5}{3} = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$

$\therefore$ Required ratio = 20 : 27

10. (B) : The inner dimensions of box are 115 cm, 75 cm and 35 cm

$\therefore$ Volume of inner box $= 115 \times 75 \times 35 = 301875$ cm^3

The outer dimensions of box are

$(115 + 2 \times 2.5)$cm, $(75 + 2 \times 2.5)$cm and $(35 + 2 \times 2.5)$cm

i.e., 120 cm, 80 cm and 40 cm

$\therefore$ Volume of outer box $= 120 \times 80 \times 40 = 384000$ cm^3

Now, Volume of wood

= Volume of outer box – Volume of inner box

$= 384000 - 301875 = 82125$ cm^3.

11. (B)

12. (C) : Radius of cone $(r) = 7$ m

Height of cone $(h) = 24$ m

$\therefore$ Now slant height of the cone, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$

$\therefore \quad l = 25$ m

Curved surface area of conical tent $= \pi rl$

$= \frac{22}{7} \times 7 \times 25 = 550$ m^2

$\therefore$ Required length of cloth

$= \frac{\text{Curved surface area of conical tent}}{\text{Width of cloth}} = \frac{550}{5} = 110$ m

13. (C) : Suppose in x hours water will be 3 metres deep in the tank.

Volume of water in the tank in x hours

$= (150 \times 100 \times 3)$ m$^3 = 45000$ m^3

Area of the cross-section of the pipe

$= \left(\frac{2}{10} \times \frac{1.5}{10}\right)$ m$^2 = \frac{3}{100}$ m^2

Volume of water that flows in the tank in x hours

= (Area of cross-section of the pipe) × (Speed of water) × (Time)

$= \left(\frac{3}{100} \times 15000 \times x\right)$ m^3 [$\because$ Speed = 15 km/hr = 15000 m/hr]

$= (450x)$ m^3

Since, the volume of water in the tank is equal to the volume of water that flows in the tank in x hours.

$\therefore \quad 450x = 45000 \Rightarrow x = 100$ hours.

14. (A) : Let the thickness of the log be x metres. Since, volume = 1.104 m^3

$\Rightarrow \quad 2.3 \times 0.75 \times x = 1.104$

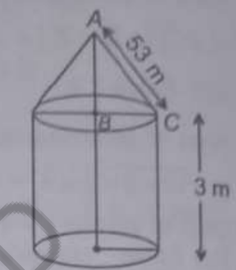
$\Rightarrow \quad x = \frac{1.104}{2.3 \times 0.75} = 0.64$ m

Since the length and breadth of each rectangular plank is the same as that of the cuboid.

$\therefore$ No. of rectangular planks

$= \frac{\text{Thickness of cuboid}}{\text{Thickness of each plank}} = \frac{0.64}{0.04} = \frac{64}{4} = 16$

15. (C) : For cylinderical part,



Radius $(r) = \frac{105}{2}$ m

Height $(h) = 3$ m

For conical part,

Slant height $(l) = 53$ m

Radius $(r) = \frac{105}{2}$ m

$\therefore$ Total curved surface area of tent $= 2\pi rh + \pi rl$

$= \pi r(2h + l) = \frac{22}{7} \times \frac{105}{2} \times (6 + 53) = (11 \times 15 \times 59)$ m^2

Hence, length of canvas

$= \frac{\text{Total curved surface area of tent}}{\text{Width of cloth}} = \frac{11 \times 15 \times 59}{5} = 1947$ m

16. (A) : Diameter of a glass = 7 cm

$\Rightarrow$ Radius of the glass $(r) = \frac{7}{2}$ cm

Height of a glass filled with milk $(h) = 12$ cm

$\therefore$ Milk contained in the cylindrical glass $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$ cm$^3 = 462$ cm^3

Now, quantity of milk required for 1600 students

$= 462 \times 1600$ cm$^3 = 739200$ cm^3

$= \frac{739200}{1000}$ litres $\quad \left[\because 1 \text{ cm}^3 = \frac{1}{1000} \text{litre}\right]$

= 739.2 litres

17. (C) : Total population of village = 5000

Water required per head per day = 75 litres

$\therefore$ Volume of water required for a small village per day

$= 5000 \times 75$ litres

$= 375000$ litres $= \frac{375000}{1000}$ m$^3 = 375$ m^3

[$\because$ 1 m^3 = 1000 litres]

Volume of an overhead tank $= (40 \times 25 \times 15)m^3 = 15000$ m^3

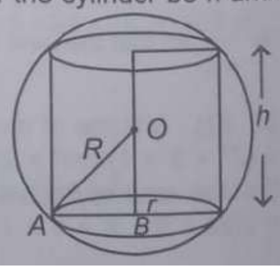
$\therefore$ Number of days $= \frac{\text{Volume of overhead tank}}{\text{Volume of water required for a small village per day}}$

$= \frac{15000}{375} = 40$

Hence, water of the tank will last for 40 days.

18. (B)

19. (C) : (P) Let height and radius of the cylinder be h and r respectively.



Let R be the radius of sphere.

In ΔOAB

$(OA)^2 = (OB)^2 + (AB)^2$

$\Rightarrow \quad R^2 = \left(\frac{h}{2}\right)^2 + r^2$

$\Rightarrow R^2 - r^2 = \frac{h^2}{4} \Rightarrow h = 2\sqrt{R^2 - r^2}$

$= 2\sqrt{(5)^2 - (3)^2} = 2 \times 4 = 8$ cm

$\therefore$ Volume of cylinder $= \pi r^2 h = \pi \times 3 \times 3 \times 8 = 72\pi$ cm^3

(Q) Radius of conical pit $(r) = \frac{3.5}{2}$ cm

Depth of conical pit (h) = 12 cm

Volume of conical pit $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 12 = 38.5$ cm^3

(R) Length of diagonal of cube of side 'a' $= \sqrt{3}a$

$\therefore \sqrt{3}a = 8\sqrt{3} \Rightarrow a = 8$ cm

Volume of cube = (side)^3 = 512 cm^3

(S) Volume of conical vessel

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (l^2 - h^2) \times 12 = \frac{1}{3} \times \pi \times ((13)^2 - (12)^2) \times 12$

$= \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi$ cm^3

20. (B) : Statement-I : Let the diameter of sphere be $2r$.

Decreased diameter $= \left(2r - \frac{25}{100} \times 2r\right) = 2r - \frac{r}{2} = \frac{3r}{2}$

New diameter $= \frac{3r}{2}$.

So, radius, $(r) = \frac{3r}{4}$

Curved surface area of sphere $= 4\pi r^2$

New curved surface area $= 4\pi\left(\frac{3r}{4}\right)^2$

Now, decreased in surface area

$= \frac{4\pi r^2 - 4\pi\left(\frac{3r}{4}\right)^2}{4\pi r^2} \times 100\% = \frac{4\pi r^2\left(1 - \frac{9}{16}\right)}{4\pi r^2} \times 100\%$

$= \frac{7}{16} \times 100\% = 43.75\%$

Statement-II : If diameter is decreased, then curved surface area is also decreased.