# Factorisation of Algebraic Expressions

**1.** The factor of $6 - y - 2y^2$ is

a $y + 2$ b $y - 3$
c $-2y + 3$ d Both (a) and (c)

**2.** After factorising $x^3 - 27$, the factors can be written in the form of

a $(x-3)(x^2+3x+9)$
b $(x+3)(x^2+3x+9)$
c $(x-3)(x^2-3x+9)$
d $(x-3)(x^2+3x-9)$

**3.** After factorising $10x^2 + 21x + 9$, we see that the factors are in the form of $(2x+3)(5x+3)$. So, the factors of 1219 can be written as, ($x$ is a natural number)

a $53 \times 33$ b $23 \times 53$
c $33 \times 23$ d None of these

**4.** It is given that $4x^2 - 12xy + 9y^2 = 0$. Then, the value of $\frac{2x}{3y}$ is

a $-1/2$ b 1
c $1/2$ d Can't say

**5.** If we factorise $xy - pq + qy - px$. The factors, thus obtained are

a $(y-p)(x+q)$ b $(y-p)(x-q)$
c $(y+p)(x+q)$ d $(y+p)(x-q)$

**6.** One of the factors of $x^4 - (x-z)^4$ is given by

a $2x + z$ b $x + 2z$
c $2x - z$ d $x - 2z$

**7.** Value of expression $\frac{58^2 - 42^2}{16}$ is

a 1 b 90 c 100 d 0

**8.** If $y^2 + 18y + 65 = ay^2 + 2by + 65$. Then, the value of $\frac{(a+b)}{(a-b)}$ is

a $\frac{19}{18}$ b $-\frac{5}{4}$
c $\frac{4}{5}$ d Can't say

**9.** If $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$ and quotient is factorised, then factors of quotient are

a $(x+1)$ b $(y-1)$
c Can't factorise d $x^2 - y^2$

**10.** Match the following:

| | List I | | List II |
|---|---|---|---|
| A. | One factor of $p^5 - 16p$ is | i. | 3 |
| B. | Real factor of $x^2 + 16$ is | ii. | Can't be calculated |
| C. | $p^3 - q^3 = (p-q) \times (\ldots)$ | iii. | $p - 2$ |
| D. | If $x^2 + \frac{1}{x^2} = 7$, then $x + \frac{1}{x}$ is | iv. | $p^2 + pq + q^2$ |

**Codes**

| | A | B | C | D |
|---|---|---|---|---|
| a | (i) | (ii) | (iii) | (iv) |
| b | (iii) | (ii) | (iv) | (i) |
| c | (iii) | (iv) | (i) | (ii) |
| d | (iii) | (iv) | (ii) | (i) |

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11. One of the factors of $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$ is given by

a $x-\frac{1}{x}$  b $x+\frac{1}{x}-1$  c $x+\frac{1}{x}-2$  d $x^2+\frac{1}{x^2}$

12. In factorisation, we use various techniques to do so. One of the techniques is illustrated here.

**Step I** $x^2-13x+42$

**Step II** $x^2-7x-6x+42$

**Step III** $x^2-13x+42+13-13$

**Step IV** $x(x-7)-6(x-7)$

**Step V** Factors are $(x-6)$ and $(x-7)$.

Which of the following steps shown above is wrong?

a I  b II  c IV  d III

13. Which method is used in the factorisation shown below?

(i) $x^2+8x+16$

(ii) $(x)^2+(4)^2+2\cdot x\cdot 4$

(iii) $(x+4)^2$ ; $(a+b)^2=a^2+b^2+2ab$

(iv) $(x+4)(x+4)$

a Splitting middle term

b Completing square

c Algebraic identity

d None of the above

14. A student is asked to factorise the expression $6x^2-30x+36$ and after factorising, he found his answers are 6, $(x+3)$ and $(x+2)$. Another student while factorising same expression found that the first student has made a mistake and after correcting it, he gave the right answer. What were the correct factors?

a $(x+3)(x-2)$  b $(x-3)(x+2)$

c $(x-3)(x-2)$  d No error

15. If $(x^2+3x+5)(x^2-3x+5)=m^2-n^2$, then $m$ is

a $x^2-3x$  b $3x$  c $x^2+5$  d $3x+5$

16. Choose the odd one from the given algebraic expressions.

(A) $x^2-4x+4$  (B) $x^2-5x+6$

(C) $x^2-9x+18$  (D) $3x^2-24x+36$

a A  b B  c C  d D

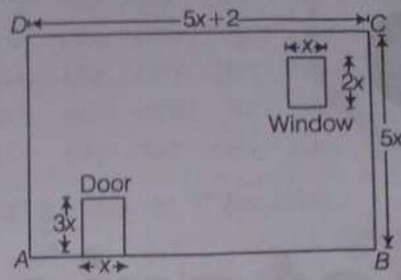
17. $\frac{0.87\times0.87\times0.87+0.13\times0.13\times0.13}{1.0}$

$=p\times(0.87)^2+q(0.87\times0.13)+r(0.13)^2$

In the given expression, what will be the value of $(p-q-r)$?

a 1  b 2  c 3  d −1

18. The figure shows, the dimensions of a wall having a window and a door at a room.




Area of wall to be painted at the rate ₹ 5 sq units. The cost charged is ₹ 50. Then, perimeter of wall is

a 10 units  b 12 units

c 14 units  d 7 units

19. Fill in the blanks with the help of options, given in the box.

(i) $2x+3$, (ii) $2x-3$, (iii) $6x^2+8x$, (iv) $8x^2+6x$, (v) 140, (vi) 110, (vii) $12x-12y$, (viii) $16x-12y$, (ix) 23, (x) 22

I. $8x^2-18x+9=(4x-3)\times$ ____.

II. $64x^4-36x^2=(8x^2-6x)\times$ ____.

III. If $x+\frac{1}{x}=5$, then $x^3+\frac{1}{x^3}=$ ____.

IV. Area of a square field is $16x^2+9y^2-24xy$. The perimeter of that field is ____.

V. If $m+n=45$ and $m^2-n^2=45$, then $m=$ ____.

**Codes**

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| a | (ii) | (iv) | (vi) | (viii) | (ix) |
| b | (i) | (ii) | (iii) | (iv) | (v) |
| c | (vi) | (vii) | (viii) | (ix) | (x) |
| d | (ii) | (iv) | (vi) | (viii) | (x) |

20. State 'T' for true or 'F' for false.

I. The factors of $x^2-6x+9$ is $(x-3)(x-3)$.

II. The difference of squares of two numbers is their sum multiplied by their difference.

III. $x^2-(a+b)x+ab=(x-a)(x+b)$

IV. Regrouping is not a method of factorisation.

V. The process of writing a given expression as the product of two or more factors is called factorisation.

**Codes**

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| a | T | T | F | F | T |
| b | T | F | T | T | T |
| c | F | F | F | T | F |
| d | T | T | F | F | F |

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1. $6 - y - 2y^2 = -(2y^2 + y - 6)$

$= -(2y^2 + 4y - 3y - 6)$

$= -\{2y(y+2) - 3(y+2)\}$

$= -\{(2y-3)(y+2)\}$

$= (y+2)(-2y+3)$

or $(2y-3)(-y-2)$

2. $x^3 - 27 = (x)^3 - (3)^3$

$= (x-3)(x^2 + 3x + 9)$

$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

3. $10x^2 + 21x + 9 = (2x+3)(5x+3)$

Put $x = 10$ in both sides, we get

$10 \cdot (10)^2 + 21 \cdot (10) + 9$

$= (2 \times 10 + 3)(5 \times 10 + 3)$

$\Rightarrow 1000 + 210 + 9 = 23 \times 53$

$\Rightarrow 1219 = 23 \times 53$

4. $4x^2 - 12xy + 9y^2 = 0$

$\Rightarrow \quad (2x)^2 + (3y)^2 - 2 \cdot 2x \cdot 3y = 0$

$\Rightarrow \quad (2x - 3y)^2 = 0$

$[\because a^2 + b^2 - 2ab = (a-b)(a-b)]$

$\Rightarrow \quad 2x - 3y = 0$

$\Rightarrow \quad 2x = 3y$

$\Rightarrow \quad \dfrac{2x}{3y} = 1$

5. $xy - pq + qy - px$

$= xy - px - pq + qy$

$= x(y-p) + q(-p+y)$

$= x(y-p) + q(y-p)$

$= (x+q)(y-p)$

6. $x^4 - (x-z)^4$

$= \{x^2\}^2 - \{(x-z)^2\}^2$

$= \{x^2 - (x-z)^2\}\{(x^2 + (x-z)^2\}$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$= \{(x - x + z)(x + x - z)\}\{x^2 + (x-z)^2\}$

$= z(2x - z)\{x^2 + (x-z)^2\}$

7. $\dfrac{58^2 - 42^2}{16} = \dfrac{58^2 - 42^2}{58 - 42}$

$= \dfrac{(58+42)(58-42)}{(58-42)} = 100$

8. $y^2 + 18y + 65 = ay^2 + 2by + 65$

On comparing, we get

$a = 1, 2b = 18 \Rightarrow b = 9$

$\therefore \quad \dfrac{a+b}{a-b} = \dfrac{1+9}{1-9} = -\dfrac{10}{8} = -\dfrac{5}{4}$

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**9.** $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$
$= (x^2y^2 + xy^2 - y^3 + 1)\, xy \div xy$
$= x^2y^2 + xy^2 - y^3 + 1$
$= xy^2(x + 1) - (y^3 - 1)$
$= xy^2(x + 1) - (y - 1)(y^2 + y + 1)$
So, we can't factorise.

**11.** $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$
$= (x)^2 + \left(\frac{1}{x}\right)^2 + 2 \cdot x \cdot \frac{1}{x} - 2\left(x + \frac{1}{x}\right)$
$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right) - 2\left(x + \frac{1}{x}\right)$
$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$

**12.** I. $x^2 - 13x + 42$
II. $x^2 - 7x - 6x + 42$
III. $x(x - 7) - 6(x - 7)$
IV. $(x - 7)(x - 6)$
∴ Step III is incorrect.

**13.** The given factorisation shows that the method of algebraic identity is being used.
i.e. $a^2 + 2ab + b^2 = (a + b)^2$
$= (a + b)(a + b)$

**14.** $6x^2 - 30x + 36$
Student found factors as $(x + 3)(x + 2)$
$= x^2 + 5x + 6$
$= 6x^2 + 30x + 36$ [multiply by 6]
But, we have
$6x^2 - 30x + 36 = 6[x^2 - 5x + 6]$
$= 6[x^2 - 3x - 2x + 6]$
$= 6[(x - 3)(x - 2)]$
∴ Factors are $(x - 3)(x - 2)$.

**15.** $(x^2 + 3x + 5)(x^2 - 3x + 5) = m^2 - n^2$
LHS $= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$
$= \{(x^2 + 5) + (3x)\}\{(x^2 + 5) - (3x)\}$
RHS $= m^2 - n^2 = (m + n)(m - n)$
Here, $m + n = (x^2 + 5) + (3x)$ [by comparing]
∴ $m = x^2 + 5$ and $n = 3x$

**16.** A. $x^2 - 4x + 4$
$= (x)^2 + (2)^2 - 2 \cdot x \cdot 2 = (x - 2)^2$
$[\because (a - b)^2 = a^2 - 2ab + b^2]$
Perfect square
B. $x^2 - 5x + 6 = x^2 - 3x - 2x + 6$
$= (x - 3)(x - 2)$
C. $x^2 - 9x + 18 = x^2 - 6x - 3x + 18$
$= (x - 6)(x - 3)$
D. $3x^2 - 24x + 36$
$= 3\{x^2 - 8x + 12\}$
$= 3\{x^2 - 6x - 2x + 12\}$
$= 3\{(x - 6)(x - 2)\}$
Except (A), all (rest) three are factorised by splitting the middle term. Also, (A) is a complete square.

**17.** $\frac{0.87 \times 0.87 \times 0.87 + 0.13 \times 0.13 \times 0.13}{0.87 + 0.13}$
$= \frac{(0.87)^3 + (0.13)^3}{(0.87 + 0.13)}$
$(0.87 + 0.13)$
$\Rightarrow \frac{\{(0.87)^2 - (0.87)(0.13) + (0.13)^2\}}{(0.87 + 0.13)}$
$= p(0.87)^2 + q(0.87 \times 0.13) + r(0.13)^2$
On comparing both sides, we get
$p = 1, q = -1, r = 1$
∴ $p - q - r = 1 - (-1) - 1$
$= 2 - 1 = 1$

**18.** Area of rectangular wall
$= 5x \times (5x + 2)$ sq units
$= 25x^2 + 10x$ sq units
Area of door $= 3x \times x = 3x^2$ sq units
Area of window $= 2x \times x$
$= 2x^2$ sq units
∴ Area to be painted = Total area − Area of (door + window)
$= 25x^2 + 10x - 3x^2 - 2x^2$
$= 20x^2 + 10x$ sq units
According to the question,
$5[20x^2 + 10x] = 50$
$\Rightarrow 2x^2 + x - 1 = 0$
$\Rightarrow 2x^2 + 2x - x - 1 = 0$
$\Rightarrow 2x(x + 1) - 1(x + 1) = 0$
$\Rightarrow (2x - 1)(x + 1) = 0$
Either $x = -1$ or $x = \frac{1}{2}$
$\Rightarrow x = \frac{1}{2}$ [neglecting −ve]
∴ Length of wall $= 5 \times \frac{1}{2} = \frac{5}{2}$
Breadth of wall $= 5 \times \frac{1}{2} + 2 = \frac{9}{2}$
∴ Perimeter $= 2(5/2 + 9/2)$
$= 14$ units

**19.** I. $8x^2 - 18x + 9 = (4x - 3) \times (2x - 3)$
II. $8x^2 + 6x$
III. $x + \frac{1}{x} = 5$ [given]
On cubing both sides, we get
$x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right) = 125$
$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 3 \times 1 \times 5$
$= 110$
IV. Area of square field
$= 16x^2 + 9y^2 - 24xy$
$= (4x)^2 + (3y)^2 - 2 \cdot 4x \cdot 3y$
$= (4x - 3y)^2$
∴ Side of field $= \sqrt{(4x - 3y)^2}$
$= 4x - 3y$
∴ Perimeter $= 4 \times (4x - 3y)$
$= 16x - 12y$
V. $\because m + n = 45$ ...(i)
and $m^2 - n^2 = 45$ [given]
$\Rightarrow (m + n)(m - n) = 45$
$\Rightarrow m - n = \frac{45}{45} = 1$ ...(ii)
From Eqs. (i) and (ii), we get
$m = 23, n = 22$

**20.** I. True II. True III. False
IV. False V. True

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